# Brabourne CEP School

## Calculation Policy – September 2017

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

# Contents



September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

# Mathematics Mastery

At Brabourne School, we believe that every child has the potential to succeed, having access to the same curriculum content as their peers, and acquiring a deep, long-term, secure and adaptable understanding of the subject. Think of it like a game of Jenga; if children learn by rote, and speed through concepts without learning them in depth, eventually, when they reach the top, there will be so many pieces missing that the foundation will not be stable and gaps will have developed in their understanding.

At Brabourne, all teachers use the White Rose Maths Hub as a process for teaching mathematics. When introducing children to new concepts, they should have the opportunity to build fluency in topics by taking the following approach:

**Concrete –** children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

**Pictorial –** children should build on this concrete approach by using pictorial representations. These representations can then be used to reason and solve problems.

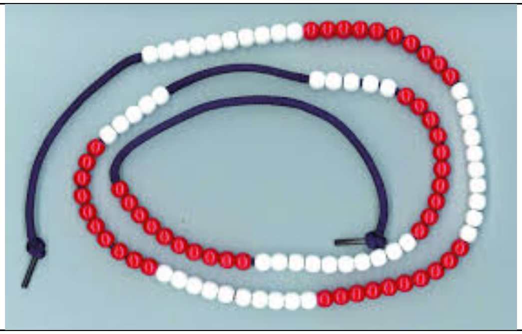
**Abstract –** with the foundations firmly laid, children should be able to move to an abstract approach using numbers and key concepts with confidence.

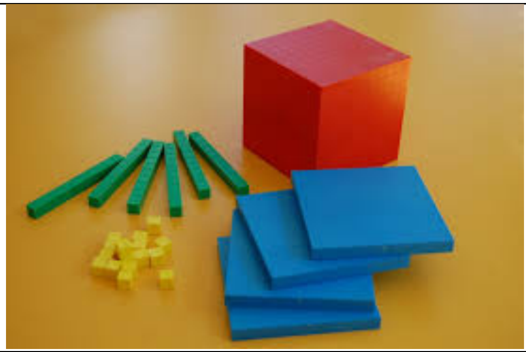
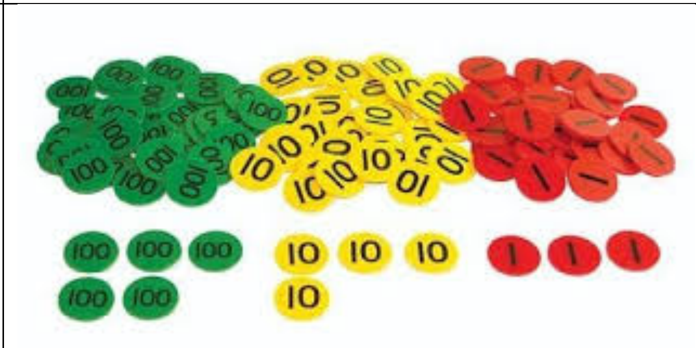
Reasoning and problem solving is encompassed in the above approaches to deepen and master all aspects of mathematics.

This policy outlines the different calculation strategies that should be taught and used from Year R to Year 6 in line with the requirements of the 2014 Primary National Curriculum, giving examples of concrete, pictorial and abstract calculations.

# Resources for Teaching Mathematics

Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept they need to master all three phases.   At Brabourne School, the following concrete objects are available for all children to use:

| | |
|---|---|
| Numicon |  |
| Bead Strings |  |

| | |
|---|---|
| Diennes (Base 10) |  |
| Cuisenaires (Bar Method) |  |
| Place Value Counters |  |

| | |
|---|---|
| Individual counters |      |

# Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

| Correct Terminology | Old Terminology (no longer used) |
|---|---|
| ones | units |
| is equal to (is the same as) | equals |
| zero | Oh (the letter o) |
| exchange<br>exchanging<br>regrouping | Stealing<br>borrowing |
| calculation<br>equation | Generic term of ‘sum’ or ‘number sentence’ |

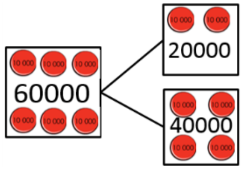
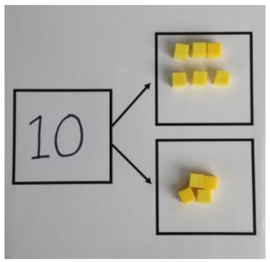
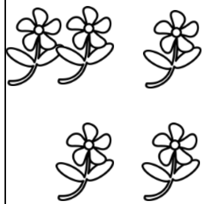
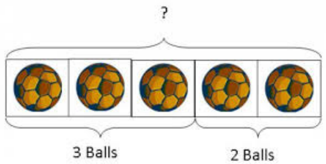
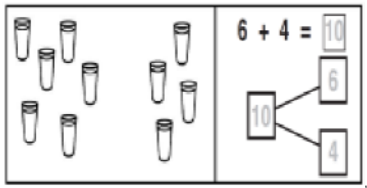
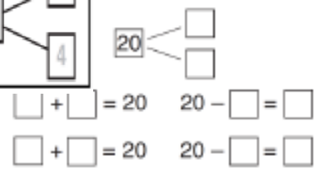
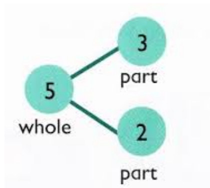
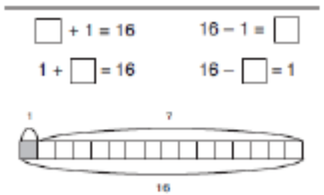
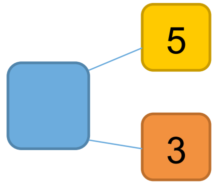
# Curriculum Objective Overview

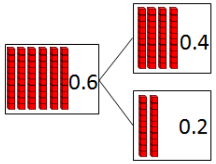
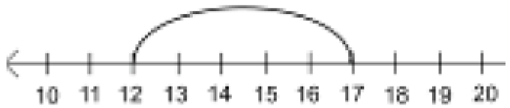
| | Year R/Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 |
|---|---|---|---|---|---|---|
| Addition | Combining two parts to make a whole; part whole model<br><br>Starting at the bigger number and counting on<br><br>Regrouping to make 10 | Adding three single digits<br><br>Column method – no regrouping | Column method regrouping (up to 3 digits) | Column method – regrouping (up to 4 digits) | Column method – regrouping (with more than 4 digits)<br><br>Decimals with the same amount of decimal places | Column method regrouping<br><br>Decimals with different amounts of decimal places |
| Subtraction | Taking away ones<br><br>Counting back<br><br>Find the different | Counting back<br><br>Find the difference<br><br>Part whole model<br><br>Make 10 | Column method with regrouping (up to 3 digits) | Column method with regrouping (up to 4 digits) | Column method with regrouping (with more than 4 digits)<br><br>Decimals with the same amount of | Column method with regrouping<br><br>Decimals with different amounts of |

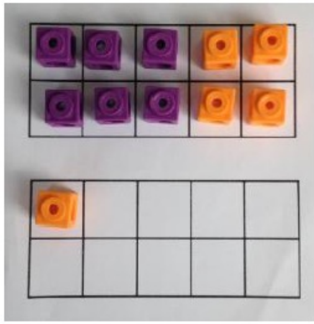
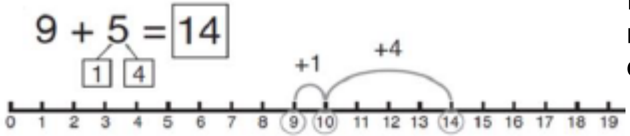
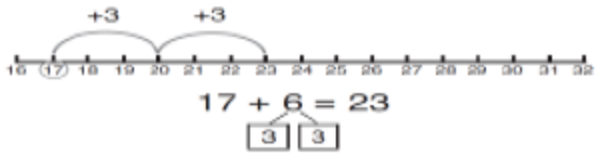
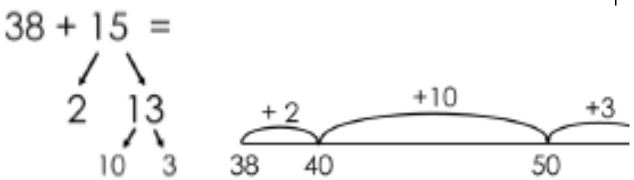
| | | | | | | |
|---|---|---|---|---|---|---|
| | Part whole model<br><br>Make 10 | Column method – no regrouping | | | decimal places | decimal places |
| Multiplication | Doubling<br><br>Counting in multiples<br><br>Arrays (with support) | Doubling<br><br>Counting in multiples<br><br>Repeated addition<br><br>Arrays – showing commutative multiplication | Counting in multiples<br><br>Repeated addition<br><br>Arrays – showing commutative multiplication<br><br>Grid method | Column multiplication (2 and 3 digits multiplied by 1 digit) | Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits) | Column multiplication (multi digit up to 4 digits by a 2 digit number |
| Division | Sharing objects into groups<br><br>Division as grouping | Division as grouping<br><br>Division within arrays | | Division within arrays<br><br>Division with a remainder<br><br>Short division (up to 3 digits by 1 digit – concrete and pictorial) | Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context | Short division<br><br>Long division (up to 4 digits by a 2 digit number<br><br>Interpret remainders as whole numbers, fractions or round |

# Strategies for the Teaching of Mastery Mathematics

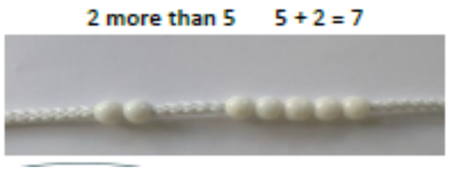
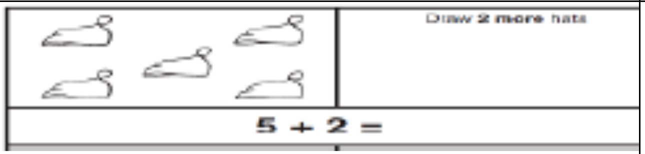
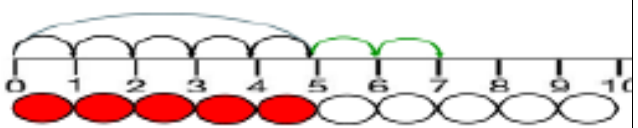
## Addition

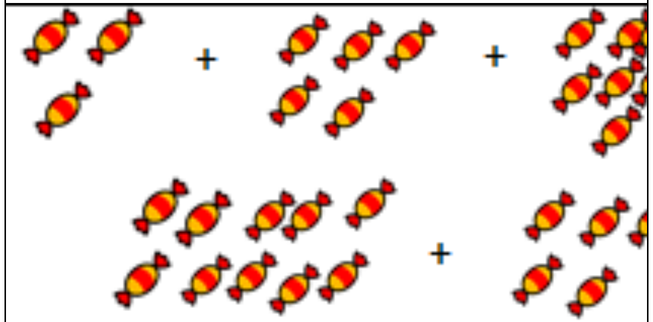
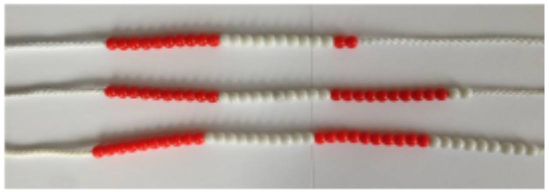
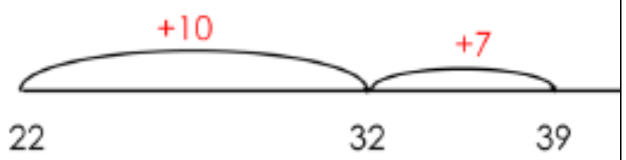
| Objective and Strategies | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Combining two parts to make a whole: part-whole model |  <br>Use cubes to add two numbers together as a group or in a bar. Then recount all using one-to-one correspondence.<br><br>Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in alongside the concrete representation.<br><br> | <br>Use pictures to add two numbers together as a group or in a bar.<br><br><br><br><br><br> | $4 + 3 = 7$<br><br>Use the part-part whole diagram as shown above to move into the abstract.<br>$10 = 6 + 4$<br>$10 - 6 = 4$<br>$10 - 4 = 6$<br>$10 = 4 + 6$ |

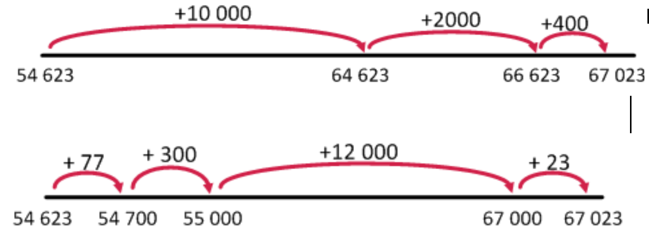
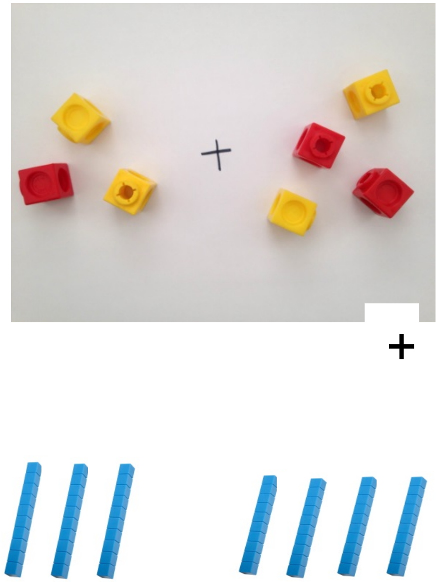
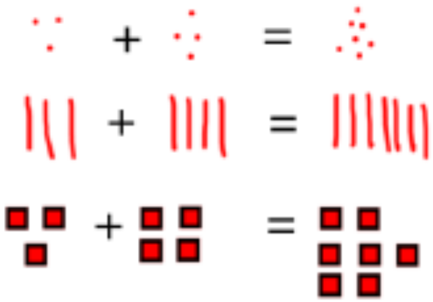
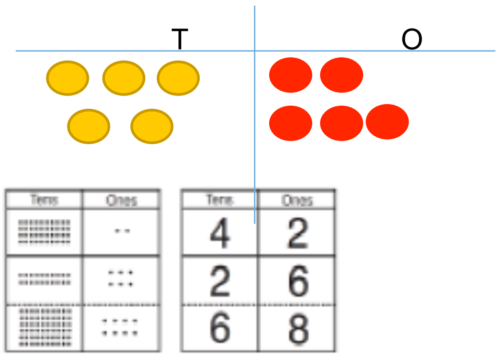
| | | | |
|---|---|---|---|
| |  | | |
| Starting at the bigger number and counting on | <br>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. | 12 + 5 = 17<br><br>Start at the larger number on the number line and count on in ones or in one jump to find the answer.<br> | 5 + 12 = 17<br>Place the larger number in your head and count on the smaller number to find your answer. |

| | | | |
|---|---|---|---|
| Make Ten Strategy |  6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. | Use pictures or a number line. Regroup or partition the smaller number to make 10.    | 7 + 4= 11<br><br>If I am at seven, how many more do I need to make 10? And how many more do I add on? |
| Regrouping (exchanging) to make 10. | The colours of the beads on the bead string make it clear how many more need to be added to make ten.   |  | NA<br><br>(This is an essential concrete/pictorial skill that will support the make ten strategy and column addition.) |

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

<table>
<tr>
<td>Adding multiples of 10</td>
<td>Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added.<br><br>50 = 30 + 20<br></td>
<td><br>3 tens + 5 tens = ___ tens<br>30 + 50 = ___</td>
<td>50 + 20 = 70<br><br>Children could count up in tens 50, 60, 70 or may recognise their number bonds 5 + 2 = 7 so 50 + 20 = 70.</td>
</tr>
<tr>
<td>Adding 1, 2, 3 more.</td>
<td>Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as;<br>‘1 more than 5 is equal to 6.’<br>‘2 more than 5 is 7.’<br>‘8 is 3 more than 5.’<br></td>
<td><br><br></td>
<td>5 + 1 = 6<br>5 + 2 = 7<br>5 + 3 = 8</td>
</tr>
</table>

| | | | |
|---|---|---|---|
| Adding three single digit numbers. | 4 + 7 + 6= 17<br>Put 4 and 6 together to make 10. Add on 7.<br><br>The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.<br>The second bead string shows 4, 6 and then 7.<br>The final bead string shows how they have now been put together to find the total. | Add together three groups of objects. Draw a picture to recombine the groups to make 10.<br> | $4 + 7 + 6 = \boxed{10} + \boxed{7}$<br>10<br>$= \boxed{17}$<br>Combine the two numbers that make 10 and then add on the remainder. |
| Partitioning one number, then adding tens and ones. | Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.<br> | <br><br>22 + 17 = 39 | 22 + 17 = 39<br><br>Counting up in head:<br>22, 32, 39. |
| Partitioning one number then counting 'on'. | Partition with cubes, dienes, bead string, counters, place value counters. | Pupils should be exposed to situations where they build an understanding of the fact that changing the order in which the parts are added or subtracted does not change the result. | Partitioning a number in their heads e,g,<br><br>1359<br><br>1 thousand<br>3 hundreds<br>five tens |

| | | | |
|---|---|---|---|
| | |  | nine ones |
| Using known facts<br><br>(I know, so…) |  | <br>3 + 4 = 7<br>*leads to*<br>30 + 40 = 70<br>*leads to*<br>300 + 400 = 7 | 5 X 4 = 20<br>So 50 x 4 = 200<br>So 50 x 40 = 2000<br>Etc.<br>Reasoning chains can be of great use to encourage children to use their known facts.<br>Using near doubles e.g.<br>2.4 + 2.5 = double 2.4 + 0.1 |
| Column method- no regrouping (exchanging) | 24 + 15=<br>Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.<br> | After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.<br> | hundreds \| tens \| ones<br>4 \| 5 \| 5<br>1 \| 0 \| 3<br>5 \| 5 \| 8<br><br>+ 26<br>68 |

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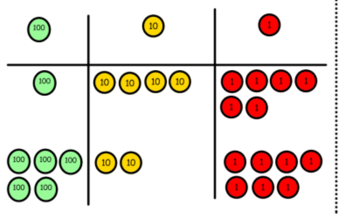
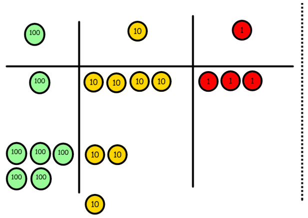
## Column method- regrouping (exchanging)

Make both numbers on a place value grid.



146
+ 527

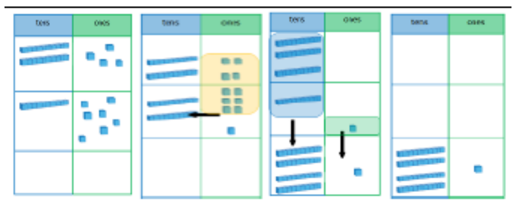
Add up the units and exchange 10 ones for one 10.



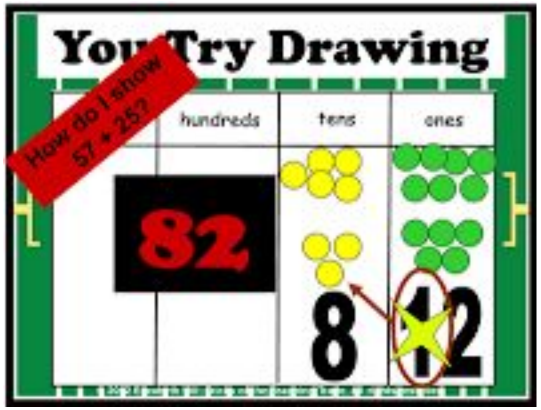
146
+ 527

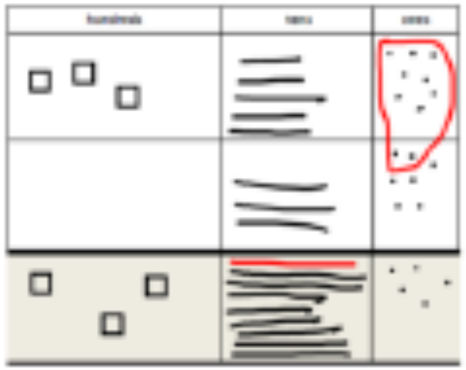
Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

As children move on to decimals, money and decimal place value counters can be used to support learning.



Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.

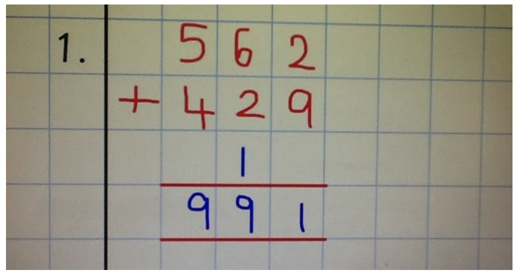


| hundreds | tens | ones |
|---|---|---|
| 3 | 5 | 8 |
| + | 3 | 7 |
| 3 | 9 | 5 |

Start by partitioning the numbers before moving on to clearly show the regrouping above the addition.

| | | | | |
|---|---|---|---|---|
| 20 | + | 5 | | |
| 40 | + | 8 | | |
| 60 | + | 13 | = | 73 |

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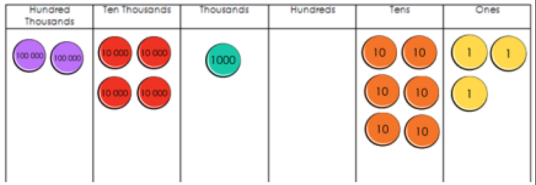
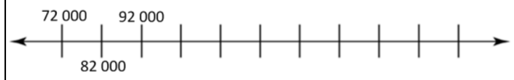
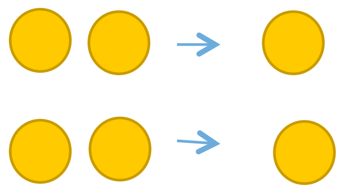
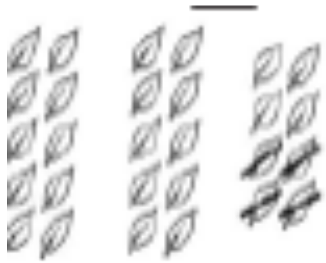
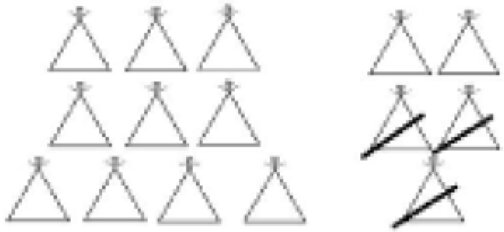
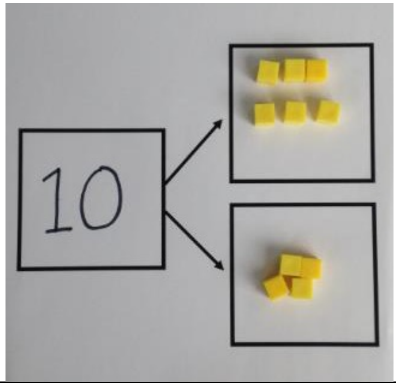
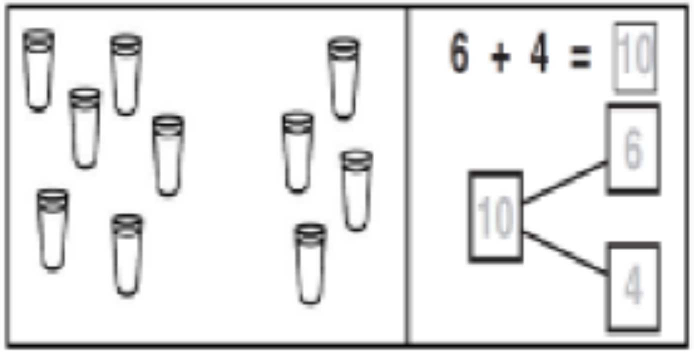
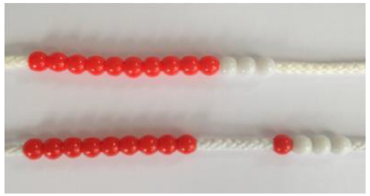
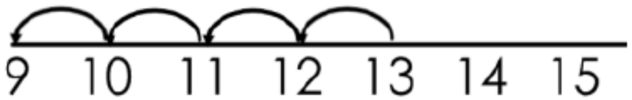
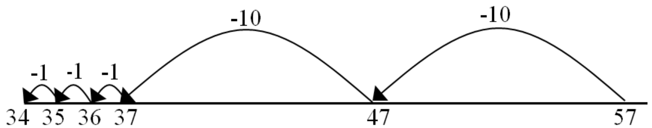
| | | | |
|---|---|---|---|
| **Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 (including tenths and hundredths – Y6)**<br>*In Year 4, pupils use place value knowledge to mentally add and subtract multiples of 10, 100 and 1000 for numbers up 10 000*<br>*In Year 5 this is extended to numbers up to 1 000 000.*<br>*In Year 6 this is extended to numbers up to 10 000 000.* | Place value grid with counters<br><br>Pay particular attention to boundaries where regrouping happens more than once. E.g. 9900 + 100 = 10000; 99 900 + 100 = 100 000; 99 000 + 1000 = 100 000 | counting stick<br> | numberline<br> |

# Subtraction

| Objective and Strategies | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Taking away ones<br><br>When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence, progressing to representing the group of ten with a tens rod and ones with ones cubes. | Use physical objects, counters, cubes etc to show how objects can be taken away.<br><br>$6 – 2 = 4$<br><br> | Cross out drawn objects to show what has been taken away.<br><br>$28 - 4 =$<br><br>$15 – 3 = \boxed{12}$ | $18 -3= 15$<br><br>$8 – 2 = 6$ |

| | | | |
|---|---|---|---|
| Part-part-whole | Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in along side the concrete representation.<br> |  | $10 = 6 + 4$<br>$10 - 6 = 4$<br>$10 - 4 = 6$<br>$10 = 4 + 6$ |
| Counting back | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.<br><br>13 – 4<br><br>Use counters and move them away from the group as you take them away counting backwards as you go.<br> | Count back on a number line or number track<br><br>Start at the bigger number and count back the smaller number showing the jumps on the number line.<br><br>This can progress all the way to counting back using two 2 digit numbers. | Put 13 in your head, count back 4. What number are you at? Use your fingers to help. |

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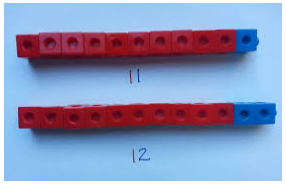
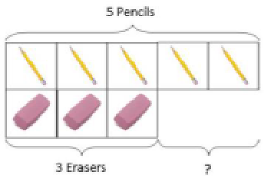
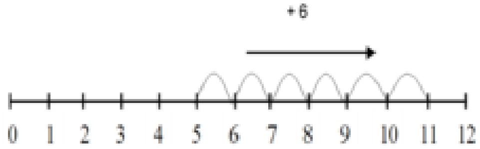
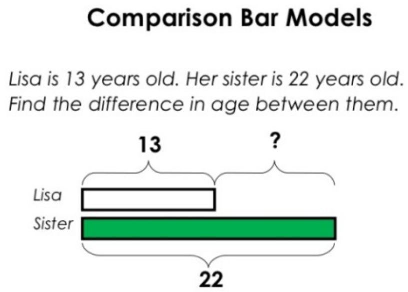
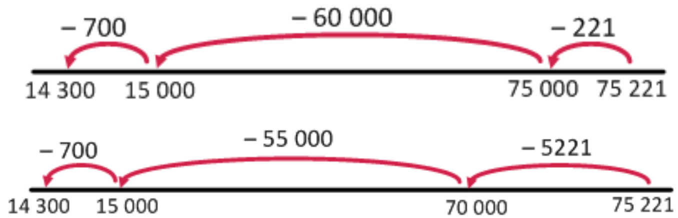
| Find the difference | Compare amounts and objects to find the difference.<br><br>Use cubes to build towers or make bars to find the difference<br><br>Use basic bar models with items to find the difference | <br>Count on to find the difference.<br>Draw bars to find the difference between 2 numbers.<br><br>Instead of subtracting or taking away, subtraction can be thought of as finding the difference between two values. Place the numbers either end of a numberline and work out the difference between them<br><br>This can be known as counting ‘on’ or ‘back’ | Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. |

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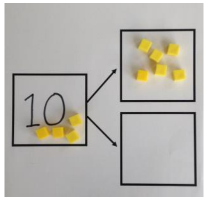
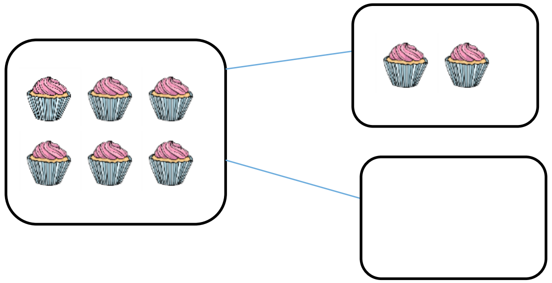
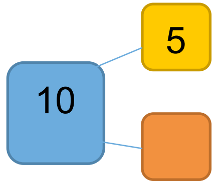
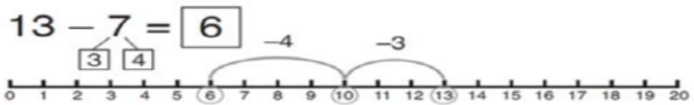
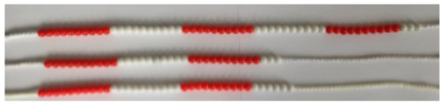
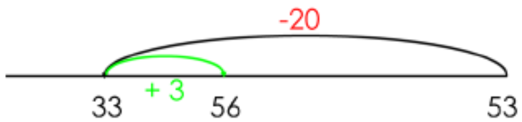
| ## Part Part Whole Model |  Link to addition- use the part whole model to help explain the inverse between addition and subtraction.<br><br>If 10 is the whole and 6 is one of the parts. What is the other part?<br><br>10 - 6 = | Use a pictorial representation of objects to show the part-part-whole model.<br> |  <br>Move to using numbers within the part whole model. |
| ## Make 10 | 14 – 9 =<br><br>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. |  <br>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. | 16 – 8=<br><br>How many do we take off to reach the next 10?<br><br>How many do we have left to take off? |
| ## Subtracting tens and adding extra ones.<br><br>Pupils must be taught to round the number that is being subtracted.<br>Pupils will develop a sense of efficiency with | <br><br>53 – 17 = 36 |  <br>53 – 17 = 36 | 53 – 17 = 36<br><br>Round 17 to 20.<br>53 – 20 = 33<br><br>20 – 17 = 3 (number bonds)<br><br>33 + 3 = 36<br>(we add because we took |

| | | | |
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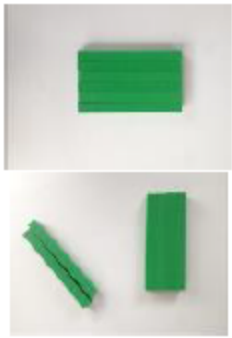
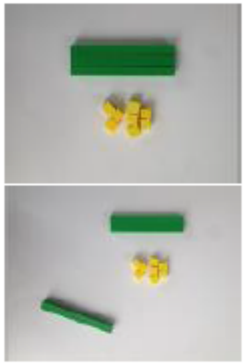
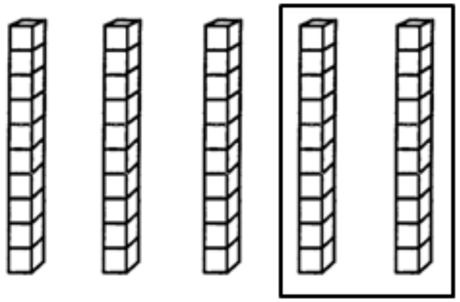
| this method, beginning to identify when this method is more efficient than subtracting tens and then ones. | | | an extra 3 away when we subtracted 20 instead of 17). |
| **Subtracting Multiples of Ten** | Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a **ten** not a one that is being taken away.<br>40 = 60 − 20<br><br>38 − 10 = 28<br> | <br>5 tens - 2 tens = ___ tens<br>50 - 20 = ____ | 32 – 10 = 22<br><br>Look at the number of tens in the largest number. Count back in tens to subtract the smaller number. 30, 20. Add on the number of ones that we originally had.  = 22 |

| | | | |
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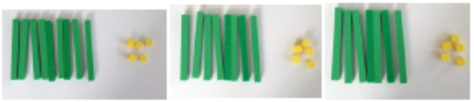
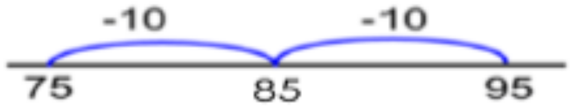
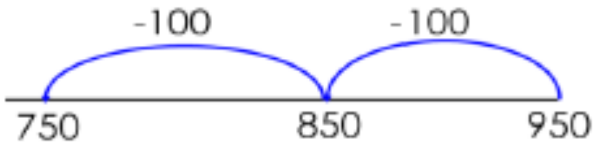
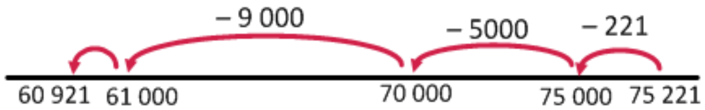
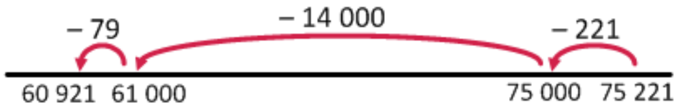
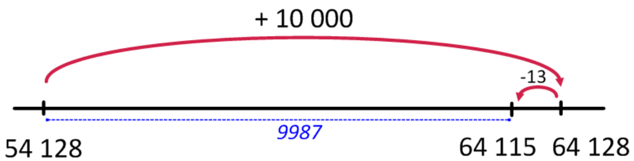
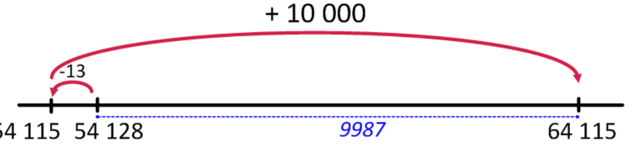
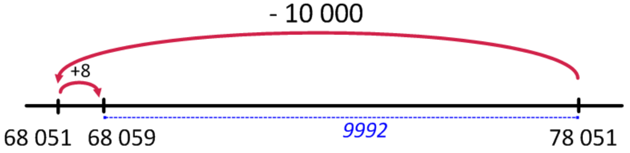
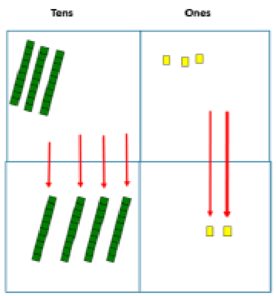
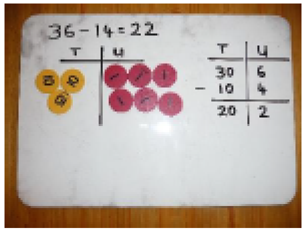
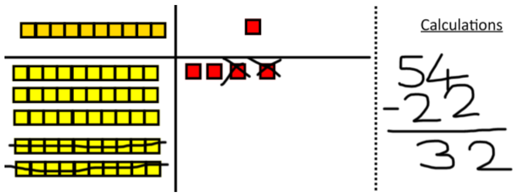
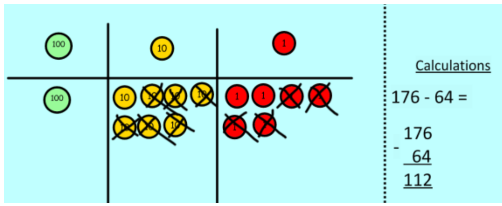
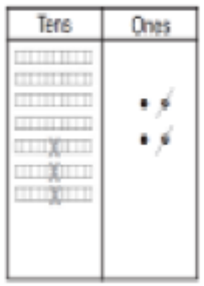
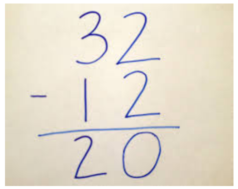
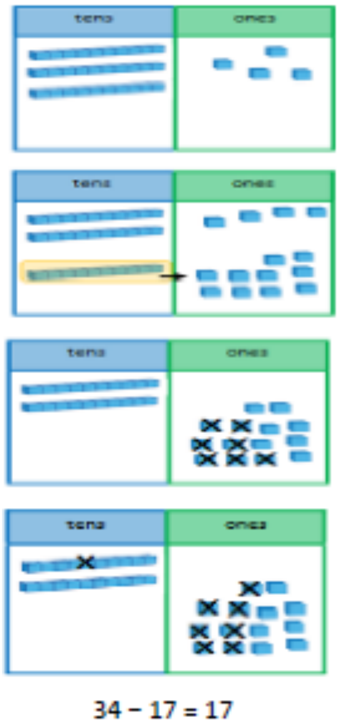
| Counting back in multiples of ten and one hundred. | <br>Removing one group of 10 each time. | <br> | Counting back in 10s or 100s from any starting point.<br><br>53, 43, 33 …<br><br>540, 440, 340 … |
| Take away | | **Parts are place value amounts (canonical partitioning)**<br><br>Pupils should understand that the parts can be subtracted in any order.<br><br>**Parts are not place value amounts (non canonical partitioning)**<br>Make ten, make hundred, make thousand, make one<br><br> | |
| Addition using compensation, rounding and adjustment | | <br>54 128 + 9987 = 54 128 + 10 000 – 13 = 64128 – 13<br>Highlight that the calculation can be done in another | Using near doubles.<br><br>E.g.<br>70 + 60 =<br><br>60 + 60 - 120 |

| | | | |
|---|---|---|---|
| | | order.<br><br><br>54128 + 9987 = 54128 – 13 + 10 000 = 54 115 + 10 000<br><br><br>78 051 – 9992 = 78 051 – 10 000 + 8 = 68 051 + 8 | 70 is 10 more than 60 so we need to add 10 more.<br><br>So the answer is 130 |
| Column method without regrouping (exchanging) | <br><br>Use Base 10 to make the bigger number then take the smaller number away.<br><br>Show how you partition numbers to subtract. Again make the larger number first.<br><br> | <br><br>Draw the Base 10 or place value counters alongside the written calculation to help to show working.<br><br><br><br> | $47 - 24 = 23$<br>$40 + 7$<br>$- \; 20 + 4$<br>$20 + 3$<br><br>This will lead to a clear written column subtraction.<br><br> |

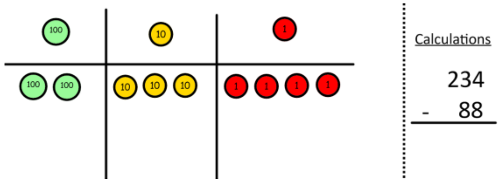
September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

## Column method with regrouping (exchanging)

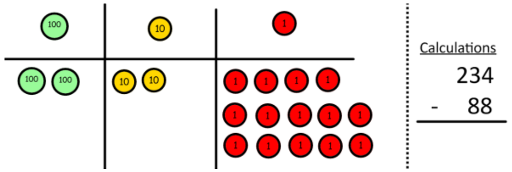



Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

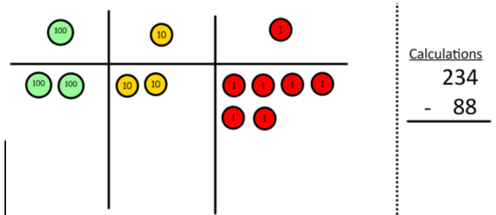
Make the larger number with the place value counters



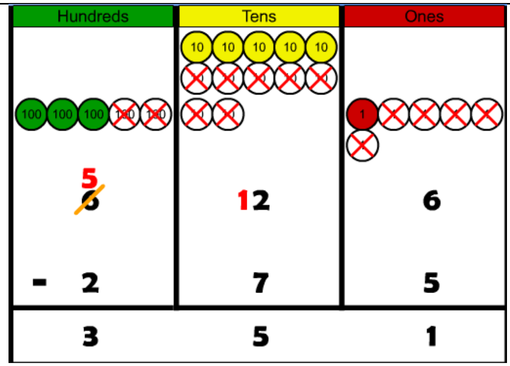

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.




Now I can subtract my ones.



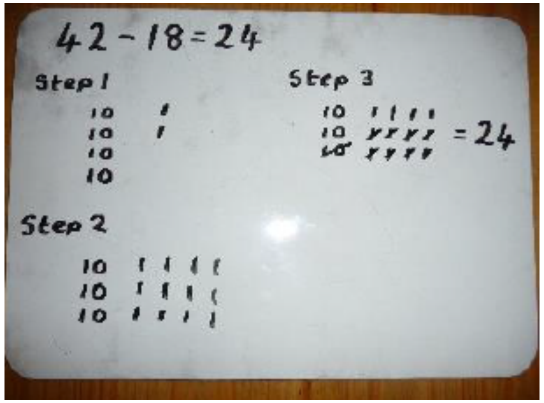

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

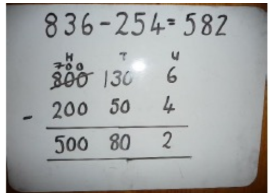



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

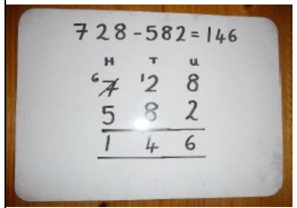
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.




Children can start their formal written method by partitioning the number into clear place value columns.




Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{rrrcr} & & {}^{5} & {}^{12} & & {}^{1} \\ & 2 & \cancel{6} & \cancel{3} & . & 0 \\ - & & 2 & 6 & . & 5 \\ \hline & 2 & 3 & 6 & . & 5 \end{array}$$

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

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<tr>
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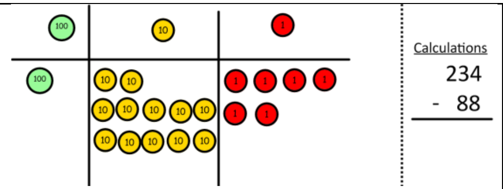
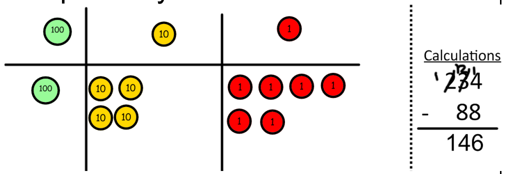


Now I can take away eight tens and complete my subtraction


Show children how the concrete method links to the written method alongside the working. Cross out the numbers when exchanging and show where we write our new amount.
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<tr>
<td>Subtraction using compensation, rounding and adjustment</td>
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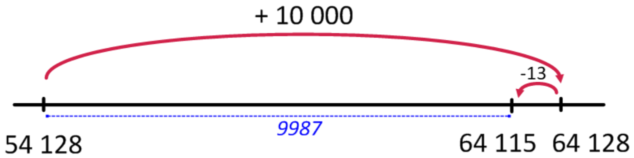
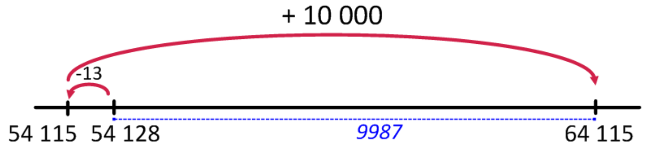
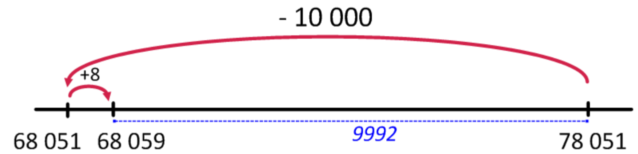
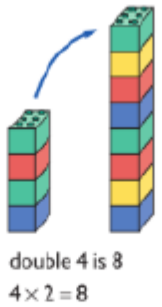
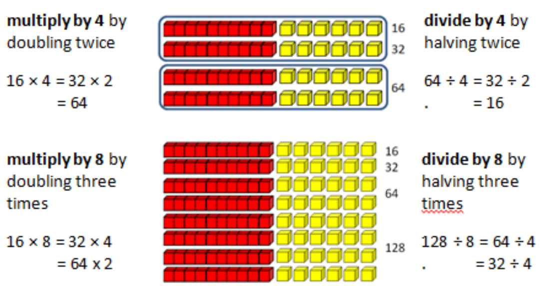
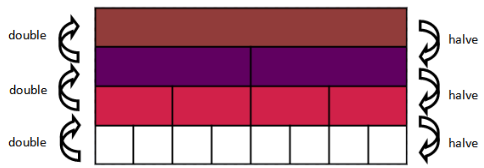
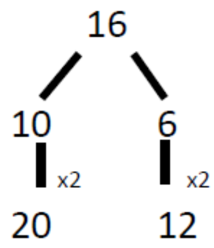


54 128 + 9987 = 54 128 + 10 000 – 13 = 64128 – 13<br>
Highlight that the calculation can be done in another order.


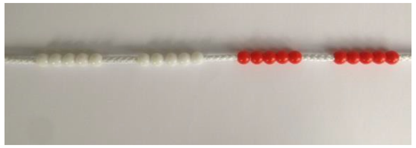
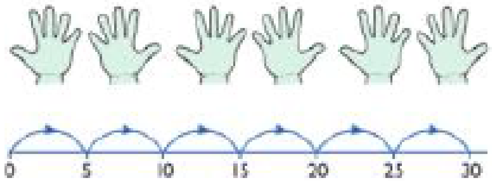
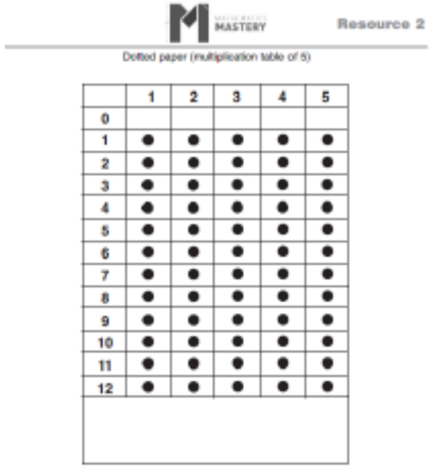
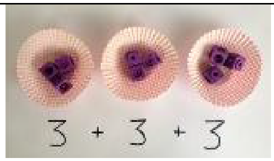
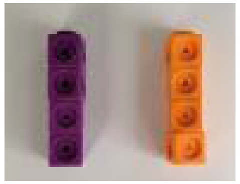
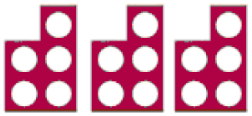
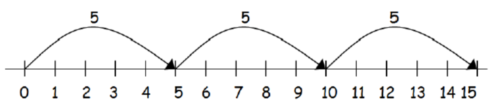
54128 + 9987 = 54128 – 13 + 10 000 = 54 115 + 10 000


78 051 – 9992 = 78 051 – 10 000 + 8 = 68 051 + 8
</td>
<td>Using near doubles.<br><br>E.g.<br>140 – 60 =<br><br>140 – 70 = 70<br><br>70 is 10 more than 60 so we need to add 10 more.</td>
</tr>
</table>

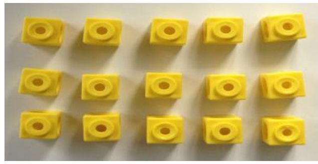
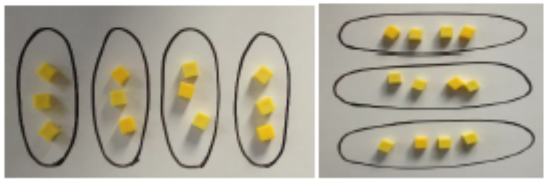
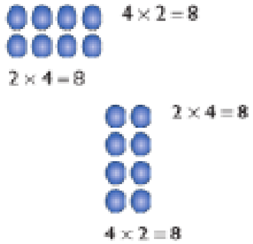
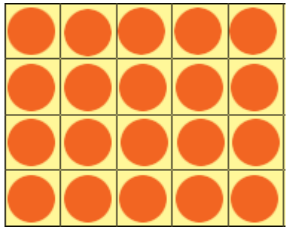
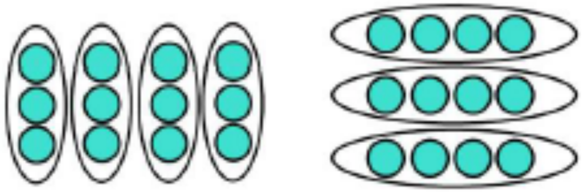
# Multiplication

| Objective and Strategies | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Doubling and halving | Use practical activities to show how to double a number.<br><br><br>Cuisenaire rods<br> | Draw pictures to show how to double a number.<br>Double 4 is 8<br> | <br>Partition a number and then double each part before recombining it back together. |

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

| | | | |
|---|---|---|---|
| Counting in multiples | <br><br>Count in multiples supported by concrete objects in equal groups. | <br><br><br><br>Use a number line or pictures to continue support in counting in multiples.<br><br>Dotted paper creates a visual representation for the different multiplication facts. | Count in multiples of a number aloud.<br><br>Write sequences with multiples of numbers.<br><br>2, 4, 6, 8, 10<br><br>5, 10, 15, 20, 25 , 30 |
| Repeated addition | <br><br><br><br>Use different objects to add equal groups. | There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?<br><br>2 add 2 add 2 equals 6<br><br><br>5 + 5 + 5 = 15<br><br>$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square$<br> | Write addition sentences to describe objects and pictures.<br><br><br>2 + 2 + 2 + 2 + 2 = 10 |

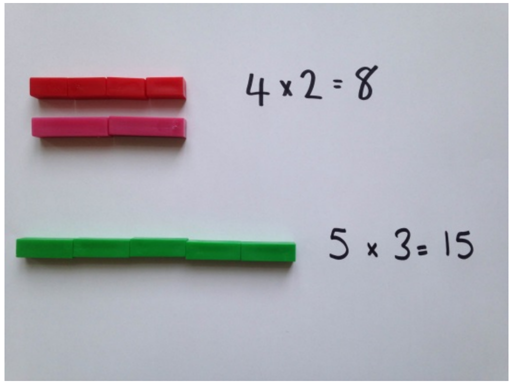
September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

| | | | |
|---|---|---|---|
| Arrays- showing commutative multiplication | Create arrays using counters/ cubes to show multiplication sentences.<br><br><br> | Draw arrays in different rotations to find **commutative** multiplication sentences.<br><br><br><br>Link arrays to area of rectangles.<br><br>12 = 3 × 4 12 = 4 × 3 | Use an array to write multiplication sentences and reinforce repeated addition.<br><br>5 + 5 + 5 = 15<br>3 + 3 + 3 + 3 + 3 = 15<br>5 x 3 = 15<br>3 x 5 = 15 |

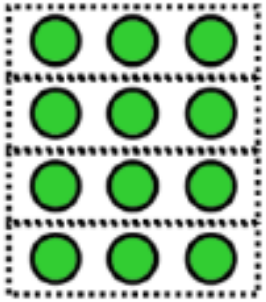
<table>
<tr>
<td>

## Bar Modelling

</td>
<td>
Cuisinaire rods can be used to create bars to represent multiplications.

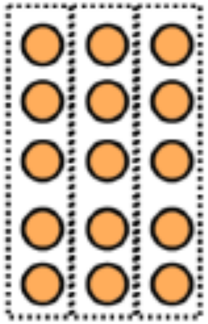
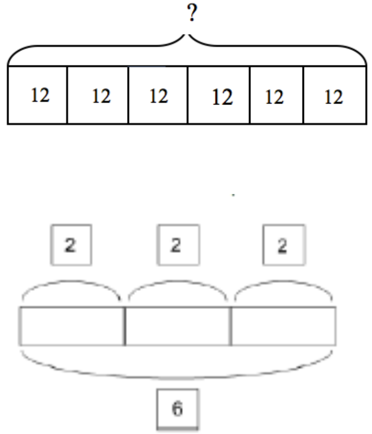


</td>
<td>
There are 4 bags of sweets with 3 sweets in each bag.
How many sweets are there altogether?



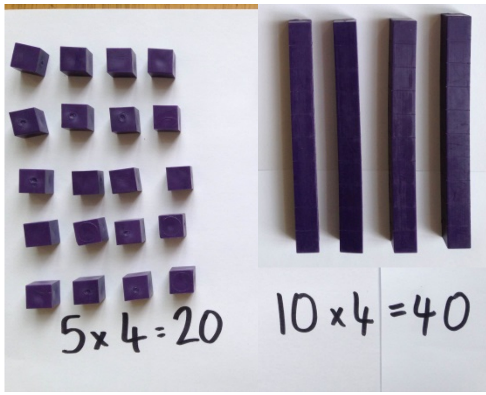
There are 3 school bags with 5 books in each one.

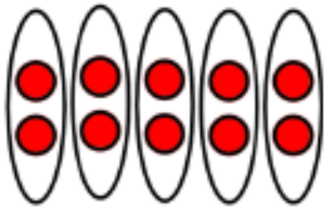
How many books are there altogether?


</td>
<td>


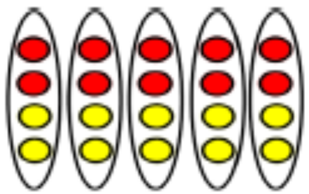
</td>
</tr>
<tr>
<td>

## Doubling to derive new multiplication facts

Pupils learn that known facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.
</td>
<td>


</td>
<td>


5 × 2 = 10

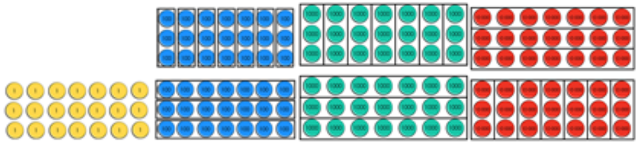


5 × 4 = 20
</td>
<td>
I know 4 x 6 = 24<br>
So, 4 x 12 = 48<br>
And 8 x 6 also = 48
</td>
</tr>
</table>

e.g. double 2x table to find 4 x table facts or double 6x5 to find 12x5 etc.

Applying the associative property allows pupils to see that this is the known fact multiplied by powers of ten. 7 x 30 = 7 x (3 x 10) = (7 x 3) x 10

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 100 000 | | 700 000 x 3 | 70 000 x 30 | 7000 x 300 | 700 x 3000 | 70 x 30 000 | 7 x 300 000 |
| 210 000 | | 70 000 x 3 | 7000 x 30 | 700 x 300 | 70 x 3000 | 7 x 30 000 | |
| 21 000 | | 7000 x 3 | 700 x 30 | 70 x 300 | 7 x 3000 | | |
| 2100 | | 700 x 3 | 70 x 30 | 7 x 300 | | | |
| 210 | | 70 x 3 | 7 x 30 | | | | |
| **21** | = | **7 x 3** | | | | | |
| 2.1 | | 0.7 x 3 | 7 x 0.3 | | | | |
| 0.21 | | 0.07 x 3 | 0.7 x 0.3 | 7 x 0.03 | | | |
| 0.021 | | 0.007 x 3 | 0.07 x 0.3 | 0.7 x 0.03 | 7 x 0.003 | | |

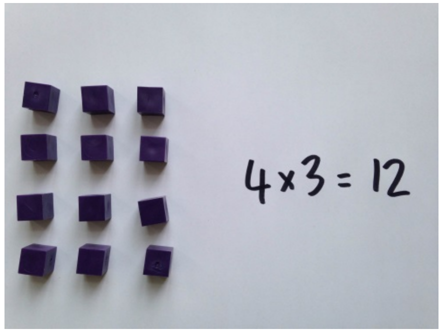


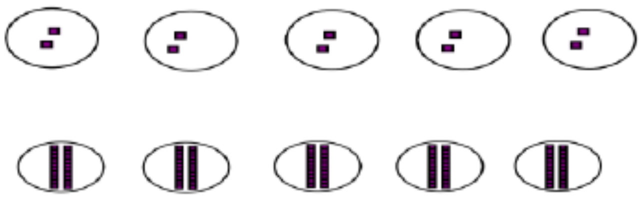
Inverse division facts can be derived:

6 ÷ 2 = 3
4 x 3 = 12
6 ÷ 3 = 2
20 x 3 = 60
1/2 ( 4 x 3 ) = 6 — 2 x 3 = 6 — 200 x 3 = 600
2 x 1 1/2 = 3
40 x 3 = 120
12 x 3 = 10 x 3 + 2 x 3 = 30 + 6 = 36

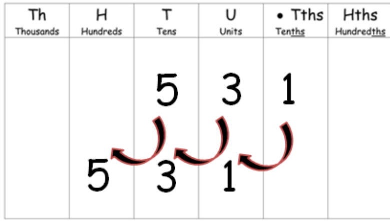
## Ten times bigger

Pupils's work on this must be firmly based on concrete representations – the language of ten times bigger must be well modelled and understood to prevent the numerical misconception of 'adding 0'.

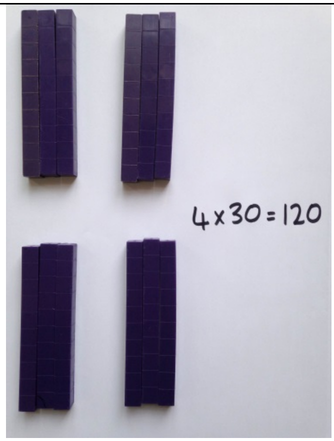


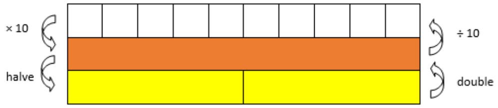
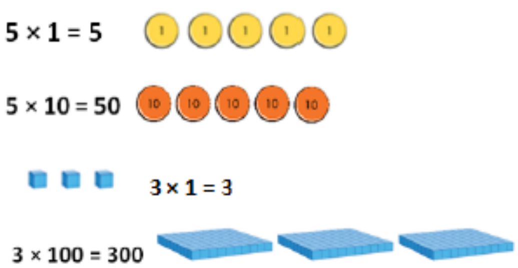



Possible misconcetion: move the deimal point.




Encouarge children to keep the decimal point stationery and move the digits once to the left in order to make the number larger and the right to make the number smaller. This should only be used when the childrne understadn the

<table>
<tr>
<td></td>
<td>

Cuisenaire rods

</td>
<td></td>
<td>concept of this through the use of using concrete and pictorial resources.</td>
</tr>
<tr>
<td>Multiplying by 10, 100 and 1000</td>
<td>
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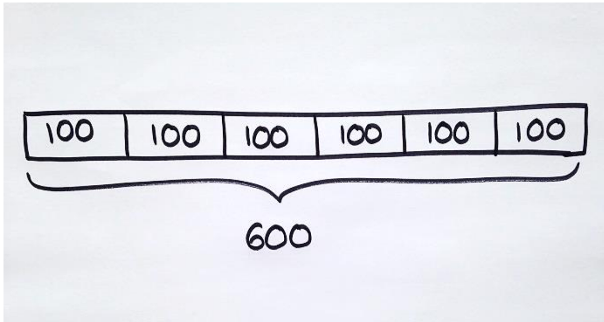
<td>
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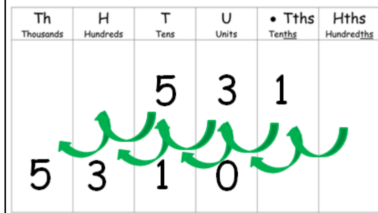
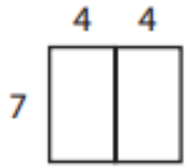
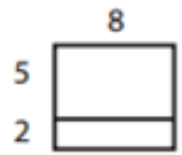
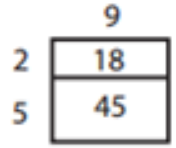
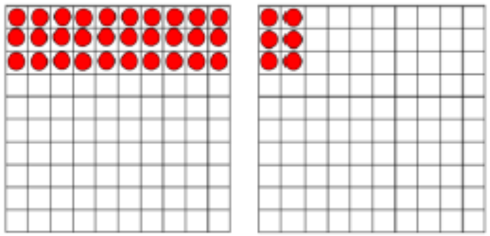
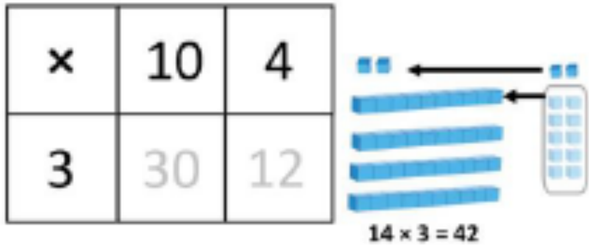
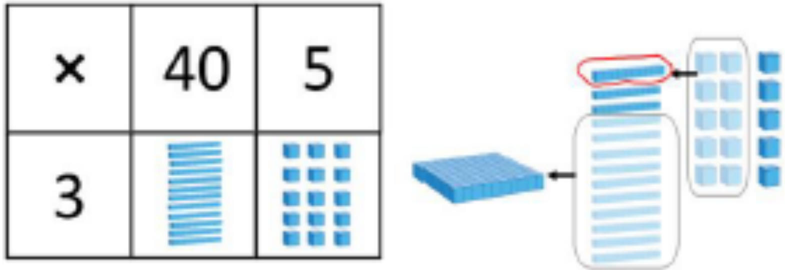
<td>
</td>
</tr>
</table>

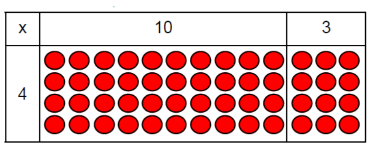
<table>
<tr>
<td>Distributive property</td>
<td>You can use dienes, counters etc. to illustrate this using arrays. Drawing out the boxes (see right) and building them up can be useful.</td>
<td colspan="2">7 x 8 is 7 x 4 and another 7 x 4:<br><br>7 x 8 is 5 x 8 and 2 x 8:<br><br><br>9 x 7 = 9 x 5 + 9 x 2</td>
</tr>
<tr>
<td>Multiplication of 2 digit numbers with partitioning (no regrouping)</td>
<td>3 x 12<br>12 = 10 + 2<br>3 X 10 3 X 2<br><br>Now add the total number of tens and ones.</td>
<td><br><table><tr><td>×</td><td>10</td><td>2</td></tr><tr><td>3</td><td>30</td><td>6</td></tr></table><br>3 x 12 = 36</td>
<td>3 x 12<br>10 and 2 make 12<br>3 x 2 = 6<br>3 x 10 = 30<br>30 + 6 = 36</td>
</tr>
</table>

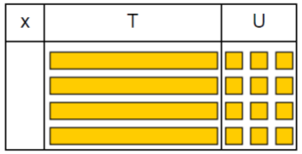
September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

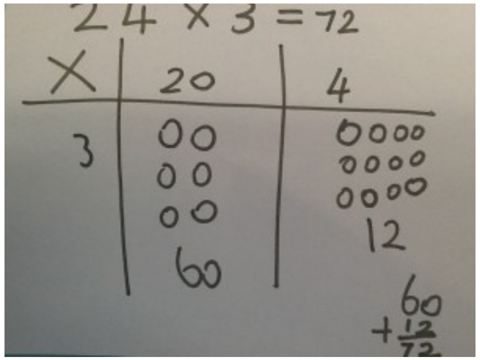
<table>
<tr>
<td>Multiplication of 2 digit numbers with partitioning (regrouping)</td>
<td colspan="2">

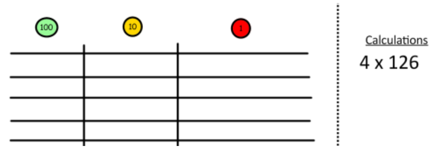


</td>
<td>3 x 25<br><br>20 and 5 make 25<br><br>3 x 5 = 15<br><br>3 x 20 = 60<br><br>And…<br>15 = 10 and 5<br><br>So…<br><br>60 + 10 = 70<br>7 + 5 = 75</td>
</tr>
<tr>
<td>Grid Method</td>
<td>Show the link with arrays to first introduce the grid method.<br><br>


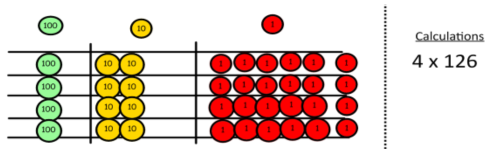
4 rows of 10<br>4 rows of 3<br><br>Move on to using Base 10 to move towards a more compact method.<br><br>


4 rows of 13<br><br>Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.<br><br>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.<br><br>


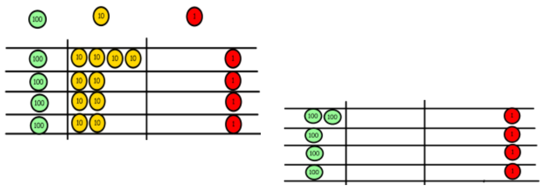
</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.<br><br>
<table>
<tr><td>×</td><td>30</td><td>5</td></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>
<br>210 + 35 = 245<br><br>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</td>
</tr>
</table>



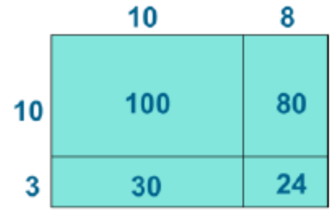

Fill each row with 126.




Add up each column, starting with the ones making any exchanges needed.



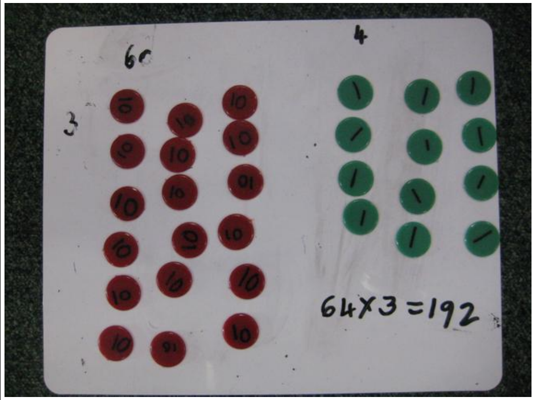
Then you have your answer.



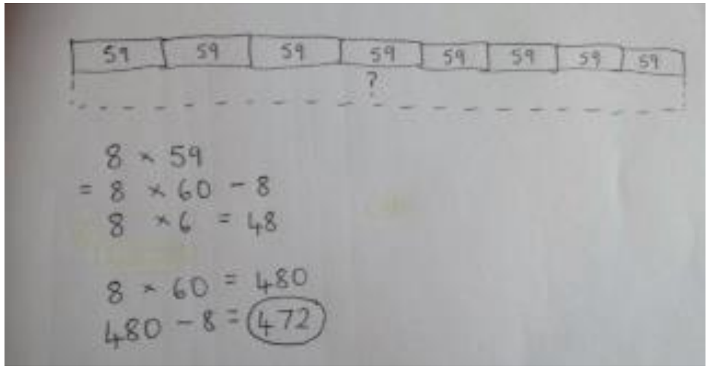

| X | 1000 | 300 | 40 | 2 |
|---|---|---|---|---|
| 10 | 10000 | 3000 | 400 | 20 |
| 8 | 8000 | 2400 | 320 | 16 |

## Short multiplication

It is important at this stage that children always multiply the ones first and note down their answer followed by the tens which they note below.



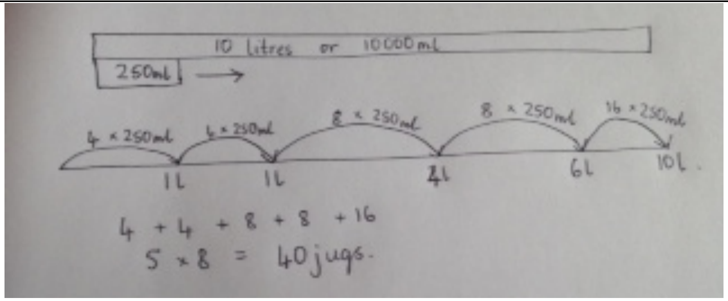

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.




Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

| | |
|---|---|
| 32 | |
| x 24 | |
| 8 | (4 x 2) |
| 120 | (4 x 30) |
| 40 | (20 x 2) |
| 600 | (20 x 30) |
| 768 | |

<table>
<tr>
<td></td>
<td>228<br>× 3<br>———<br></td>
<td>
</td>
<td>74<br>× 63<br>———<br>12<br>210<br>240<br>+ 4200<br>———<br>4662<br><br>This moves to the more compact method.<br><br>(carries: 2 3 1)<br>1342<br>x 18<br>———<br>13420<br>10736<br>———<br>24156<br>(carry: 1)</td>
</tr>
</table>

National Curriculum appendix:

**Long multiplication**

24 × 16 becomes

| | | | |
|---|---|---|---|
| | | 2 | |
| | | **2** | **4** |
| **×** | | **1** | **6** |
| | **2** | **4** | **0** |
| | **1** | **4** | **4** |
| | **3** | **8** | **4** |

Answer: 384

124 × 26 becomes

| | | | | |
|---|---|---|---|---|
| | | 1 | 2 | |
| | | **1** | **2** | **4** |
| **×** | | | **2** | **6** |
| | **2** | **4** | **8** | **0** |
| | | **7** | **4** | **4** |
| | **3** | **2** | **2** | **4** |
| | 1 | 1 | | |

Answer: 3224

124 × 26 becomes

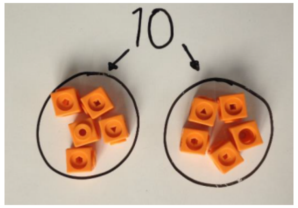
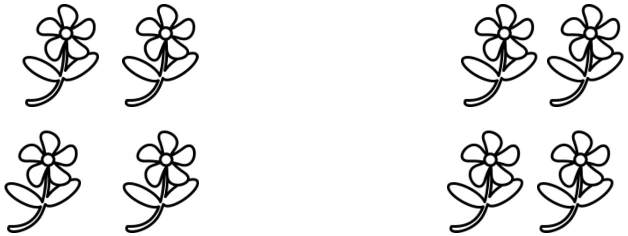
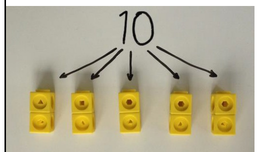
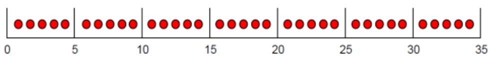
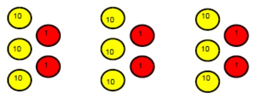
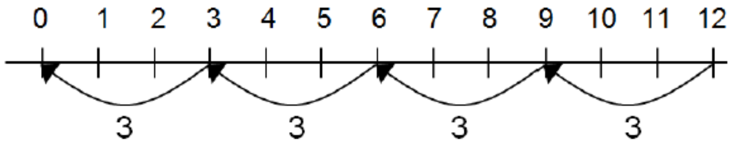
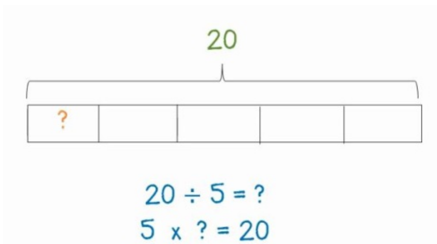
| | | | | |
|---|---|---|---|---|
| | | 1 | 2 | |
| | | **1** | **2** | **4** |
| **×** | | | **2** | **6** |
| | | **7** | **4** | **4** |
| | **2** | **4** | **8** | **0** |
| | **3** | **2** | **2** | **4** |
| | 1 | 1 | | |

Answer: 3224

Note: Use a line in between the final factor and the answer for exhanging, rather than in the pictures to the left.

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

# Division

| Objective and Strategies | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Sharing objects into groups |  <br><br><br>I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities.<br><br>$\boxed{8 \div 2 = 4}$ | Share 9 buns between three people.<br><br>$9 \div 3 = 3$ |
| Division as grouping | Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.<br><br><br><br><br>$96 \div 3 = 32$<br><br> | Use a number line to show jumps in groups. The number of jumps equals the number of groups.<br><br><br>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.<br><br> | $28 \div 7 = 4$<br><br>Divide 28 into 7 groups. How many are in each group? |

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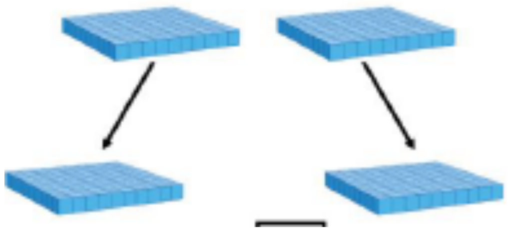
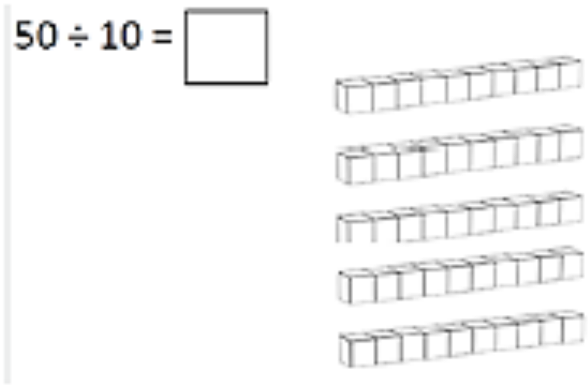
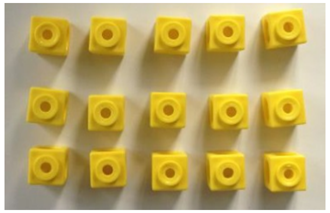
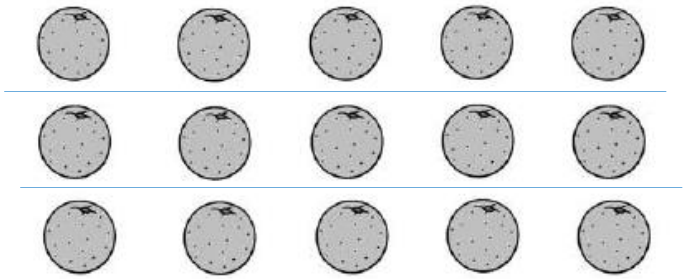
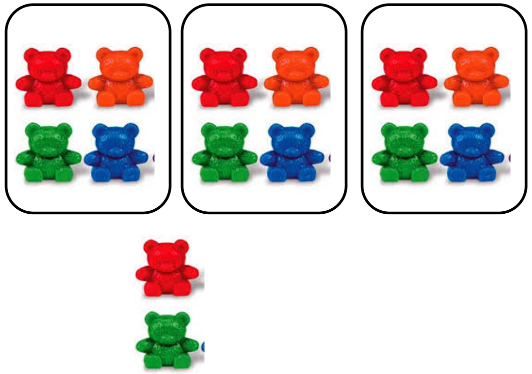
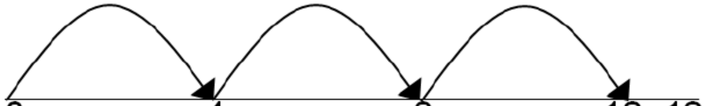
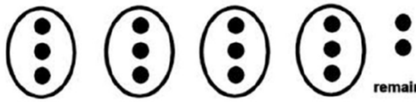
| | | | |
|---|---|---|---|
| **Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000.**<br><br>Pupils use the strategy of sharing into equal groups of tens, hundreds or thousands to reinforce their understanding of place value and the concept of division as sharing into equal groups. They master this skill with calculations where no partitioning is required, to prepare them for the next step | <br><br>200 ÷ 100 = 2<br><br>Here the child has selected the 100 dienes to use because they're dividing by 100. So 200 divided into groups of 100 = 2 groups. |  | 6000 ÷ 200 = 30<br><br>"I know there are five groups of 200 in 1000 and I have six 1000s and 5 x 6 = 30." |
| **Division within arrays** | <br>Link division to multiplication by creating an array and thinking about the number sentences that can be created.<br><br>Eg 15 ÷ 3 = 5 5 x 3 = 15<br>15 ÷ 5 = 3 3 x 5 = 15 | <br>Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating four linking number sentences.<br><br>7 x 4 = 28<br>4 x 7 = 28<br>28 ÷ 7 = 4<br>28 ÷ 4 = 7 |

| | | | |
|---|---|---|---|
| Division with a remainder | 14 ÷ 3 =<br>Divide objects between groups and see how much is left over<br> | Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.<br><br><br>Draw dots and group them to divide an amount and clearly show a remainder.<br><br> | Complete written divisions and show the remainder using r.<br>29 ÷ 8 = 3 REMAINDER 5<br>dividend divisor quotient remainder |

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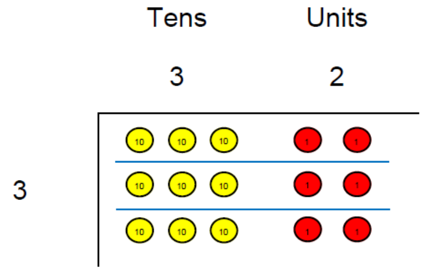
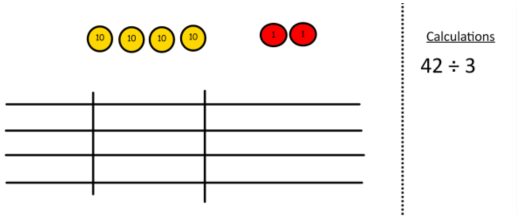
# Short division

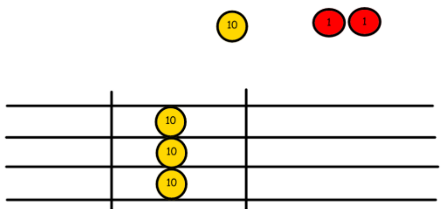
The difficulty with the short division algorithm comes with the confusion that can be caused by what you "think in your head" The thought process of the traditional algorithm is as follows:
How many 4s in 8? 2
How many 4s in 5? 1 with 1 remaining so regroup.
How many 4s in 12? 3
How many 4s in 8? 2
Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.
How many 4s in 500? 100 with 1 remaining (illogical) The answer would be 125
Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.
Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the compact short division algorithm.



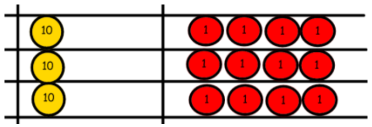

Use place value counters to divide using the bus stop method alongside




42 ÷ 3=
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

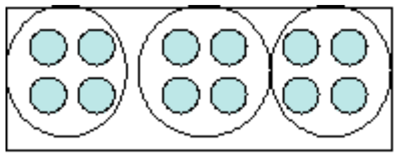


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much there is in 1 group; the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally.

2 1 8
3
4 | 8 7 2

Move onto divisions with a remainder.

8 6 r 2
3
5 | 4 3 2

Finally move into decimal places to divide the total accurately.

1 4 . 6
16 21
3 5 | 5 1 1 . 0

See below for written strategies:

September 2017. Images and ideas drawn from White Rose Maths Hubs, Sandgate Primary School and Beckford Primary School.

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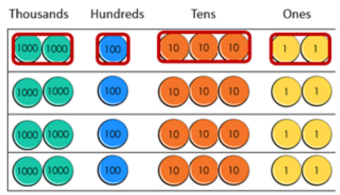
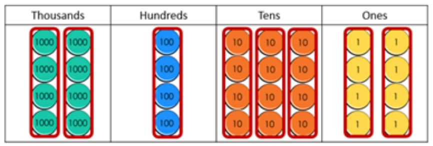
```
    2 1 3 2
4 | 8 5¹2 8
```

**Sharing**




Share 8528 into 4 equal groups.<br>
8 thousands shared into 4 equal groups<br>
5 hundreds shared into 4 equal groups<br>
Regroup 1 hundred for 10 tens<br>
12 tens shared into 4 equal groups<br>
8 ones shared into 4 equal groups

**Grouping**




How many groups of 4 in 8528?<br>
How many groups of 4 thousands in 8 thousands?<br>
How many groups of 4 hundreds in 5 hundreds?<br>
Regroup 1 hundred for 10 tens<br>
How many groups of 4 tens in 12 tens?<br>
How many groups of 4 ones in 8 ones?

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Written version of a mental strategy for 3-digit ÷ 1 digit numbers

Short division of 3-digit and 4-digit numbers by single-digit numbers

```
   1  2  6  4
6 | 7 ¹5 ³8 ²4
```

$\square \times 6 = 326$     $326 \div 6 = 54\ r2$

$50 \times 6 = \underline{300}$<br>
26<br>
$4 \times 6 = \underline{24}$<br>
2<br>
54 r2

</td>
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## Long Division

</td>
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The short division method can be applied for 11 and 12 using times tables knowledge. Factors shoul dbe used to break down the calculation and apply the short division method. If the divisor is a print number see opposite.

</td>
<td>

```
      2 1 2
13 | 2 7 5 6
     2 6
       1 5
       1 3
         2 6
         2 6
           0
```

</td>
<td>

```
      2 1 2
13 | 2 7 5 6
     2 6 0 0
       1 5 6
       1 3 0
         2 6
         2 6
           0
```

</td>
</tr>
</table>

National Curriculum appendix:

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## Short division

98 ÷ 7 becomes

| | | |
|---|---|---|
| | 1 | 4 |
| 7 | 9 | $^{2}$8 |

Answer: 14

432 ÷ 5 becomes

| | | | | |
|---|---|---|---|---|
| | | 8 | 6 | r 2 |
| 5 | 4 | 3 | $^{3}$2 | |

Answer: 86 remainder 2

496 ÷ 11 becomes

| | | | | | |
|---|---|---|---|---|---|
| | | | 4 | 5 | r 1 |
| 1 | 1 | 4 | 9 | $^{5}$6 | |

Answer: $45\frac{1}{11}$

## Long division

432 ÷ 15 becomes

| | | | | | |
|---|---|---|---|---|---|
| | | | 2 | 8 | r 12 |
| 1 | 5 | 4 | 3 | 2 | |
| | | 3 | 0 | 0 | |
| | | 1 | 3 | 2 | |
| | | 1 | 2 | 0 | |
| | | | 1 | 2 | |

432 ÷ 15 becomes

| | | | | | |
|---|---|---|---|---|---|
| | | | 2 | 8 | |
| 1 | 5 | 4 | 3 | 2 | |
| | | 3 | 0 | 0 | 15×20 |
| | | 1 | 3 | 2 | |
| | | 1 | 2 | 0 | 15×8 |
| | | | 1 | 2 | |

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

432 ÷ 15 becomes

| | | | | | |
|---|---|---|---|---|---|
| | | | 2 | 8 · | 8 |
| 1 | 5 | 4 | 3 | 2 · | 0 |
| | | 3 | 0 | ↓ | |
| | | 1 | 3 | 2 | |
| | | 1 | 2 | 0 | ↓ |
| | | | 1 | 2 | 0 |
| | | | 1 | 2 | 0 |
| | | | | | 0 |